

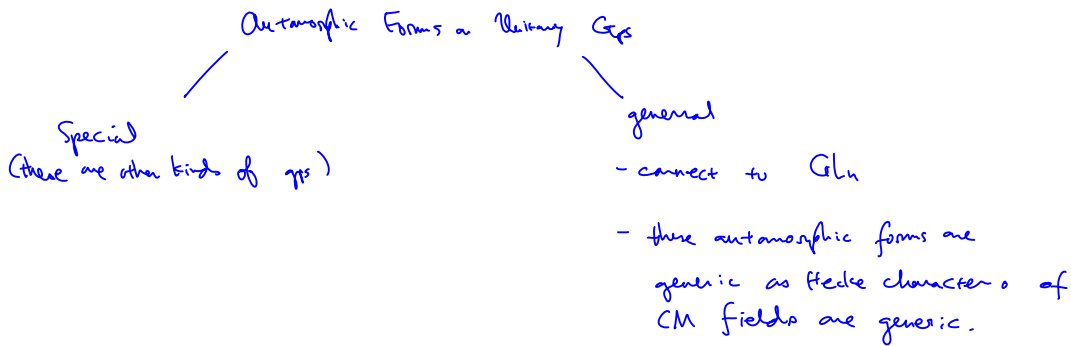
Constructing p -adic L -functions

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Relate p -adic L -functions to characteristic function of Selmer groups.

First step: constructing p -adic L -functions

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K quad. imag. fld

$$c \in \text{Gal}(K/\mathbb{Q})$$

W/K Hermitian space of dim n

$\langle u, v \rangle$ linear in first variable, c -linear in second variable.

hermitian symm.

$-W$ space W , \langle, \rangle is $-\langle, \rangle$.

$2W = W \oplus (-W)$ simplest kind of hermitian space.

$$\| W \oplus W^* \quad \langle (w, w), (v, v) \rangle = 0 \quad \forall v, w \in W.$$

totally isotropic subspaces

$$H = U(2W)$$

$$G = U(W)$$

$P = \text{Stab}_H(W^*)$
max. parabolic subgp.

$$m(A) = \begin{pmatrix} A & \\ & cA^{-1} \end{pmatrix} \in P$$

$$M = \{m(A)\}$$

$$P \cong \begin{pmatrix} A & X \\ 0 & c(A^{-1}) \end{pmatrix}$$

R a G algebra

$$U(R) = \left\{ \begin{pmatrix} 1 & X \\ & 1 \end{pmatrix} \in P \right\}$$

X runs over $\text{Hom}_n(R \otimes K)$.

$$A \in GL_n(K).$$

$$A = A \oplus \mathbb{Q}$$

$$w = \begin{pmatrix} 0 & I_n \\ -I_n & 0 \end{pmatrix}$$

I. Construction of p -adic L -funs

II. p -adic L -fun of (nearly ordinary Hida families)

doublet formula for L -functions of unitary reps
 generalization of Katz's construction in the case $n=1$.

Consider the subgrps of H with values in \mathbb{Q} , completion of \mathbb{Q} , \mathbb{A} .

Let v be a completion of \mathbb{Q} ,

$$\delta_v(p) = |N_{\mathbb{Z}/\mathbb{Q}} \circ \det A(p)|^{\frac{h}{2}} \quad (\text{modulus function})$$

$\delta_{\mathbb{A}}$ adelic version $\prod \delta_v$

$\chi: \mathbb{Z}_{\mathbb{A}}^{\times} / \mathbb{Z}^{\times} \rightarrow \mathbb{C}^{\times}$ Hecke character.

χ is chara. of \mathbb{A}^{\times} by composition with det of $\text{GL}_n(\mathbb{A})$.

$$\delta_{\mathbb{A}}(p, \chi, s) = \delta_{\mathbb{A}}(p) \cdot \chi \circ \det(A(p)) \cdot |N_{\mathbb{Z}/\mathbb{Q}} \circ \det(A(p))|_{\mathbb{A}}^s$$

$$\sum_{\gamma \in P(\mathbb{Q}) \backslash H(\mathbb{Q})} \delta_{\mathbb{A}}(p(\gamma h), \chi, s) = E(\chi, \chi, s) \quad \mathbb{Z}_{\mathbb{A}}$$

$$H(\mathbb{Q}) = \text{PGL}_n(\mathbb{Q}) \cdot K_{\mathbb{A}}$$

$K_{\mathbb{A}} =$ adelic max. compact.

$h = p(h) \cdot \gamma(h)$
not well defined.

Eisenstein series

General Eisenstein series attached to functions (non-normalized induction)

$$f(\chi, \chi, s) \in \text{Ind}_{\text{PGL}_n(\mathbb{Q})}^{H(\mathbb{Q})} \delta(\cdot, \chi, s) = \left\{ f: H(\mathbb{Q}) \rightarrow \mathbb{C} \mid f(\gamma h) = \delta(p, \chi, s) f(\chi, h) \right\}$$

$$\sum_{\gamma \in P(\mathbb{Q}) \backslash H(\mathbb{Q})} \frac{y^s}{|cz+d|^{2s}} E_f(\chi, \chi, s) = \sum_{\gamma \in P(\mathbb{Q}) \backslash H(\mathbb{Q})} f(\gamma h, \chi, s) \quad \text{converges for } \text{Re } s \gg 0.$$

For appropriate choices of f and s

$$n=1, \quad H = \text{U}(1,1) \cup \text{SL}_2$$

$E_f|_{\text{SL}_2(\mathbb{R})}$ is essentially
 appropriately $s \leftrightarrow k$

$$y^{-\frac{k}{2}} \sum \frac{1}{(cz+d)^2}$$

$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \dots$$